Hash-based signatures II

Stateful and stateless signatures

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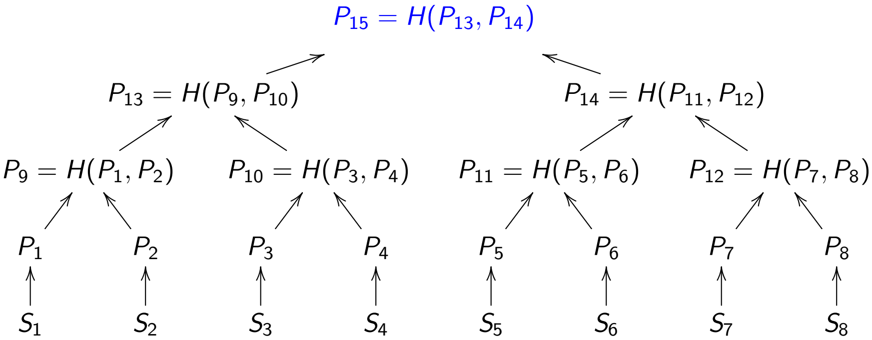
Merkle's (e.g.) 8-time signature system

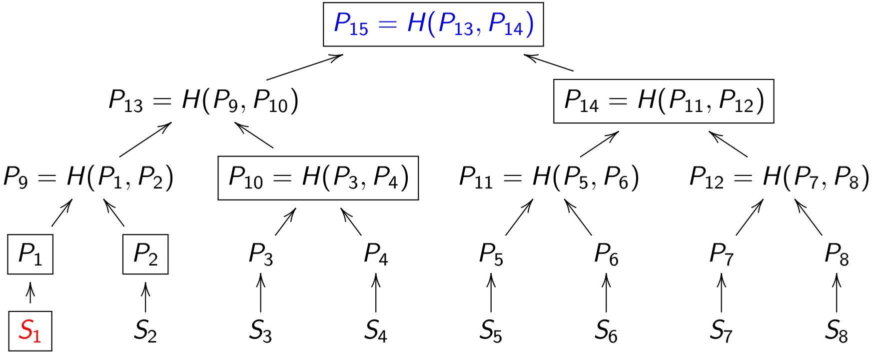
Hash 8 one-time public keys into a single Merkle public key P_{15}.



$S_i \to P_i$ can be Lamport or Winternitz one-time signature system.
Each such pair (S_i, P_i) may be used only once.

Signature in 8-time Merkle hash tree

Signature of first message: $(\texttt{sign}(m, S_1), P_1, P_2, P_{10}, P_{14})$.



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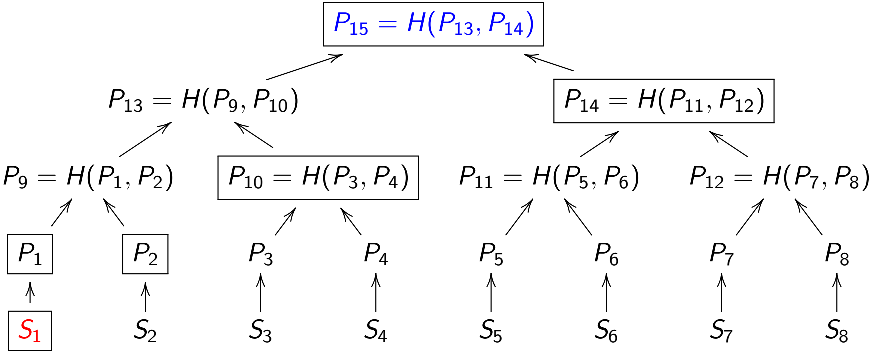

Verify signature $\texttt{sign}(m, S_1)$ with public key P_1 (provided in signature).
Link P_1 against public key P_{15} by computing $P'_9 = H(P_1, P_2)$,
$P'_{13} = H(P'_9, P_{10})$, and comparing $H(P'_{13}, P_{14})$ with P_{15}.
Reject if $H(P'_{13}, P_{14}) \neq P_{15}$ of if the signature verification failed.

Signature in 8-time Merkle hash tree

Signature of sixth message:

$$
\begin{tikzcd}[column sep=small]
 & & & \boxed{P_{15} = H(P_{13}, P_{14})} & & & & \\
 & P_{13} = H(P_9, P_{10}) \arrow[urr] & & & & P_{14} = H(P_{11}, P_{12}) \arrow[ull] & & \\
P_9 = H(P_1, P_2) \arrow[ur] & & P_{10} = H(P_3, P_4) \arrow[ul] & & P_{11} = H(P_5, P_6) \arrow[ur] & & P_{12} = H(P_7, P_8) \arrow[ul] & \\
P_1 \arrow[u] & P_2 \arrow[ul] & P_3 \arrow[u] & P_4 \arrow[ul] & P_5 \arrow[u] & P_6 \arrow[ul] & P_7 \arrow[u] & P_8 \arrow[ul] \\
S_1 \arrow[u] & S_2 \arrow[u] & S_3 \arrow[u] & S_4 \arrow[u] & S_5 \arrow[u] & \boxed{S_6} \arrow[u] & S_7 \arrow[u] & S_8 \arrow[u]
\end{tikzcd}
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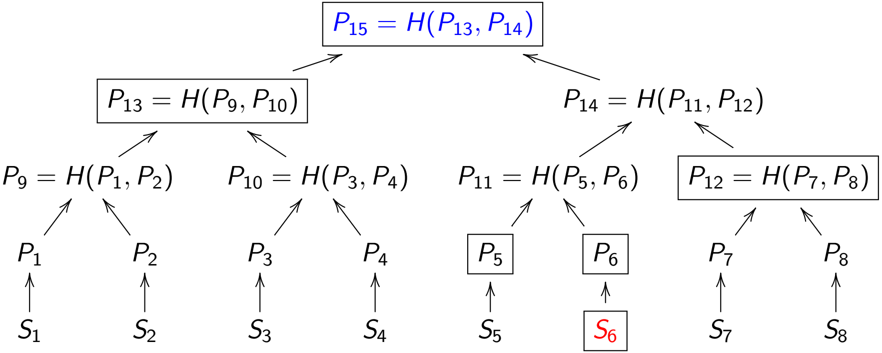

Improvements to Merkle's scheme

- Each public key (root of the tree) is good only for fixed number of messages, typically 2^n.
- The public key is very short: just one hash output.
  But each signature contains n public keys along with the one-time signature.
- Computing the public key requires computing and storing 2^n one-time signature keys.

Trees of Merkle trees

$$P_{15} = H(P_{13}, P_{14})$$

$$P_{13} = H(P_9, P_{10}) \qquad P_{14} = H(P_{11}, P_{12})$$

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$$P_1 \qquad P_2 \qquad P_3 \qquad P_4 \qquad P_5 \qquad P_6 \qquad P_7 \qquad P_8$$

$$S_1 \qquad S_2 \qquad S_3 \qquad S_4 \qquad S_5 \qquad S_6 \qquad S_7 \qquad S_8$$

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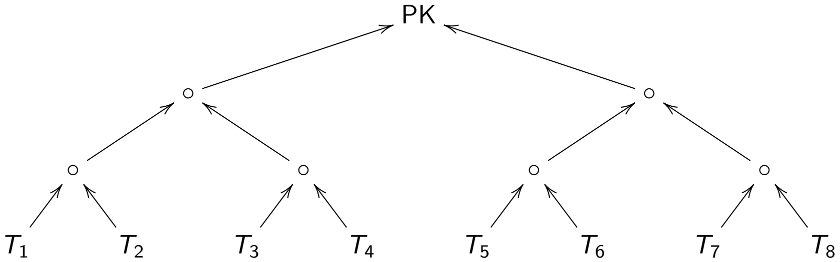

T_i are one-time signature keys.
$\uparrow$ indicates input to hash function.

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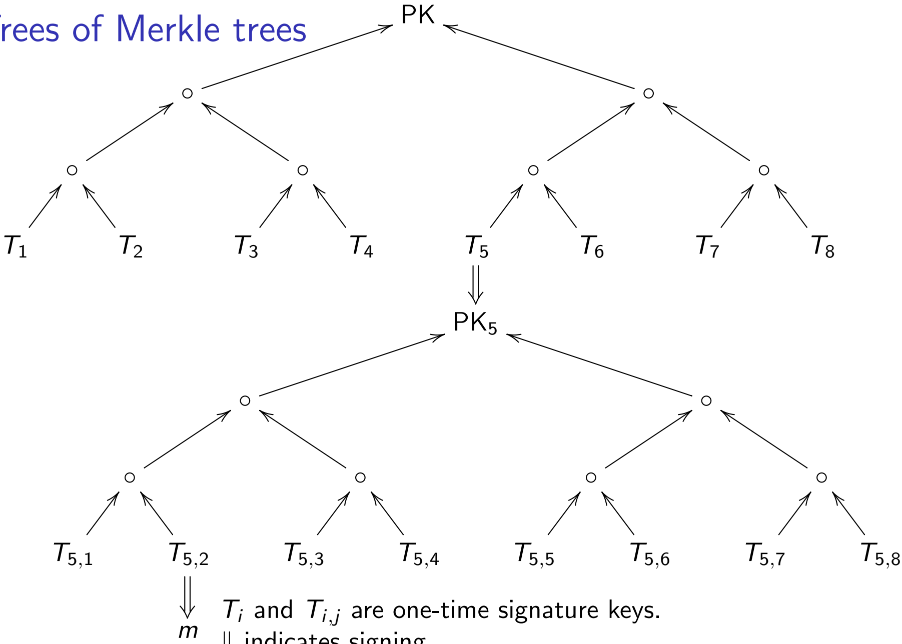

T_i and $T_{i,j}$ are one-time signature keys.
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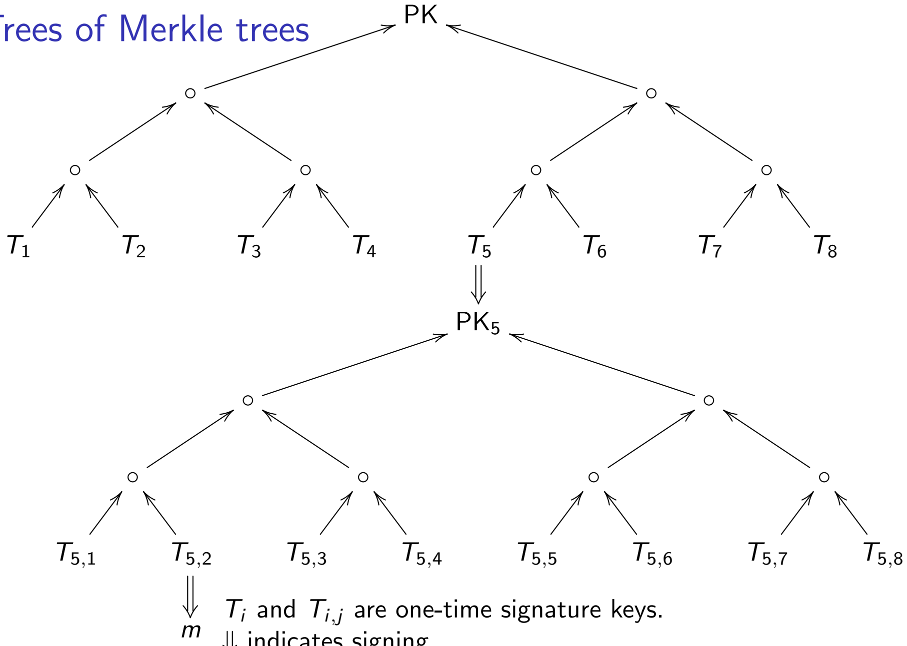

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No need to know PK_5 when generating the top tree.

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- Building trees of trees increases the signature length (one extra one-time signature per tree) and signing time. See PhD thesis of Andreas Hülsing for an optimized schedule of what to store and when to precompute.
  Only the top tree is needed to generate the public key.

Stateful hash-based signatures

- Only one prerequisite: a good hash function, e.g. SHA3-512.
  Hash functions map long strings to fixed-length strings.
  Signature schemes use hash functions in handling plaintext.
- Old idea: 1979 Lamport one-time signatures.
- 1979 Merkle extends to more signatures.

Pros:

- Post quantum
- Only need secure hash function
- Security well understood
- Fast

Cons:

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  though some tradeoffs possible
- Stateful, i.e., ever reusing a subkey breaks security.
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- We can count: OS update, code signing, ... naturally keep state.

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Standardization progress

- CFRG has published 2 RFCs: RFC 8391 and RFC 8554

Datatracker Groups Documents Meetings Other User

```
Internet Research Task Force (IRTF)                          A. Huelsing
Request for Comments: 8391                                  TU Eindhoven
Category: Informational                                         D. Butin
ISSN: 2070-1721                                             TU Darmstadt
                                                               S. Gazdag
                                                              genua GmbH
                                                            J. Rijneveld
                                                     Radboud University
                                                             A. Mohaisen
                                           University of Central Florida
                                                                May 2018

                XMSS: eXtended Merkle Signature Scheme
```

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```
Internet Research Task Force (IRTF)                    D. McGrew
Request for Comments: 8554                             M. Curcio
Category: Informational                               S. Fluhrer
ISSN: 2070-1721                                    Cisco Systems
                                                      April 2019

          Leighton-Micali Hash-Based Signatures
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- ISO SC27 JTC1 WG2 is working on standard for stateful hash-based signatures.

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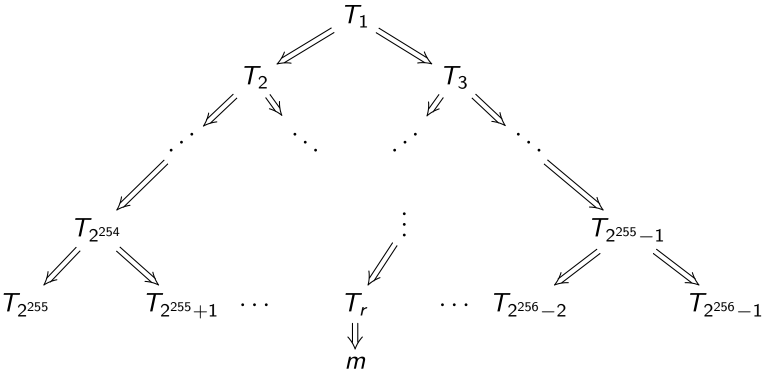
- Can we build trees so large that this is not a problem?
- By the birthday paradox we need 2^{256} leaves!
- Cannot precompute this tree ...

Huge trees (1987 Goldreich), keys on demand (Levin)

Signer chooses random $r \in \{2^{255}, 2^{255}+1, \ldots, 2^{256}-1\}$,
uses one-time public key T_r to sign message;
uses one-time public key T_i to sign (T_{2i}, T_{2i+1}) for $i < 2^{255}$.
Generates ith secret key deterministically as $H_k(i)$ where k is master secret. Important for efficiency



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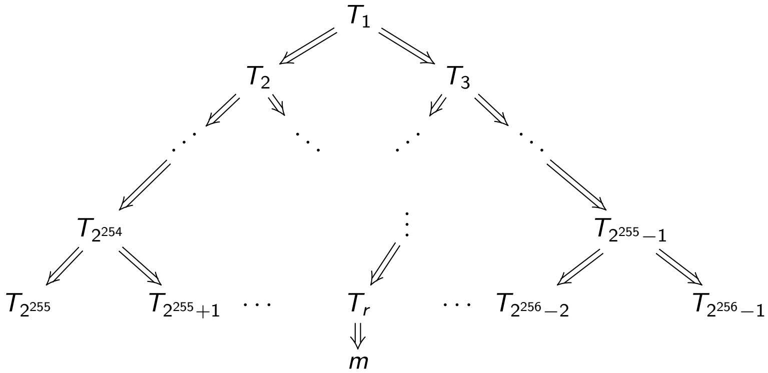

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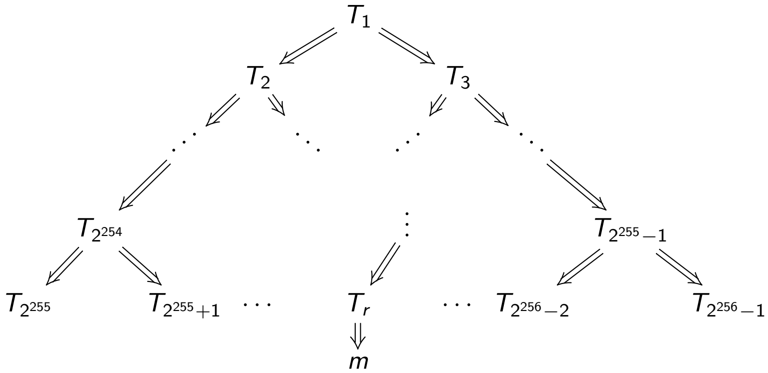

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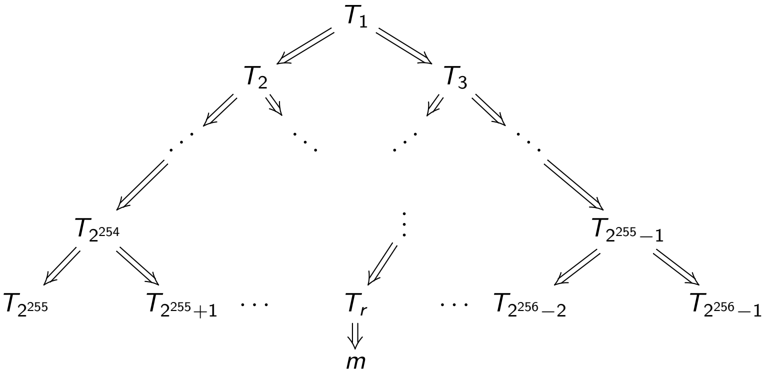

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Use Goldreich to create stateless hash-based signatures

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Example:
HTTPS typically sends multiple signatures per page.
1.8 MB average web page in Alexa Top 1000000.

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Change definition of H to have many components

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where each $h_i \in \{0, 1, 2, \ldots, t-1\}$ for some t.
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r-subset resilience

Let $H(m_j) = (h_{j,0}, h_{j,1}, \ldots, h_{j,k-1})$.
H is r-subset-resilient if given $H(m_1), H(m_2), \ldots, H(m_r)$
the probability of finding $m' \neq m_i$ with $H(m') = (h'_0, h'_1, \ldots, h'_{k-1})$ and
$h_f \in \{h_{j,i} | 0 \leq i < k, 1 \leq j \leq r\}$ for $0 \leq f < k$ is negligible.

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The same leaf public key can be used for $r+1$ signatures
if H if r-subset-resilient.

Few-times signature HORS

(Hash to Obtain Random Subset)

General parameters:

- Integer parameters k, t, ℓ.
- Hash function $H : \{0,1\}^* \to \{0,1\}^{k \cdot \log_2 t}$.
- One-way function $f : \{0,1\}^\ell \to \{0,1\}^\ell$.

KeyGen:

- Picks t strings $s_i \in \{0,1\}^\ell$, compute $v_i = f(s_i)$ for $0 \leq i < t$.
- Public key $P = (v_0, v_1, \dots, v_{t-1})$; secret key $S = (s_0, s_1, \dots, s_{t-1})$.

Sign $m \in \{0,1\}^*$:

- Compute $H(m) = (h_0, h_1, \dots, h_{k-1})$, where each $h_i \in \{0, 1, 2, \dots, t-1\}$.
- Signature on m is $\sigma = (s_{h_0}, s_{h_1}, s_{h_2}, \dots, s_{h_{k-1}})$.

Verify:

- Compute $H(m) = (h_0, h_1, \dots, h_{k-1})$ and $(f(s_{h_0}), f(s_{h_1}), f(s_{h_2}), \dots, f(s_{h_{k-1}}))$.
- Verify that $f(s_{h_i}) = v_{h_i}$ for $0 \leq i < t$.

HORS exercises, assume H is surjective

1. Let $\ell = 80$, $t = 2^5$, and $k = 3$. How large (in bits) are the public and secret keys? How large is a signature? How many different signatures can the signer generate for a fixed key pair as $H(m)$ varies? Ignore that s-values could collide.
2. The same public key can be used for $r + 1$ signatures if H is r-subset-resilient.
   Even for $r = 1$, i.e. after seeing just one typical signature, an attacker has an advantage at creating a fake signature. What are the options beyond exact collisions in H?
3. Let $\ell = 80$, $t = 2^5$, and $k = 3$. Let m be a message so that $H(m) = (h_0, h_1, h_2)$ satisfies that $h_i \neq h_j$ for $i \neq j$. You get to specify messages that Alice signs. You may not ask Alice to sign m. State the smallest number of HORS signatures you need to request from Alice in order to construct a signature on m? How many calls to H does this require on average? You should assume that H and f do not have additional weaknesses beyond having too small parameters. Explain how you could use under 1000 evaluations of H if you are allowed to ask for two signatures.

Ingredients of SPHINCS (and SPHINCS-256)

Drastically reduce tree height (to 60).

Replace one-time leaves with few-time leaves.

Optimize few-time signature size *plus* key size.
New few-time HORST (HORS with trees),
improving upon HORS.

Use hyper-trees (12 layers), as in GMSS.

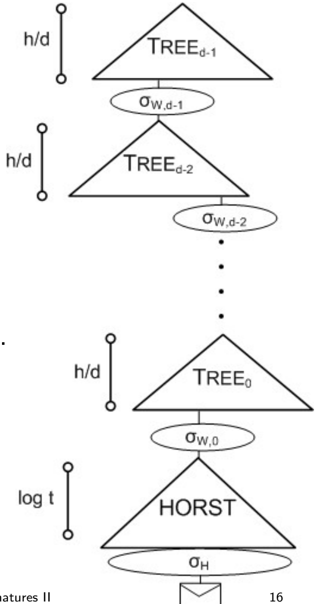
Use masks, as in XMSS and XMSS^{MT},
for standard-model security proofs.

Optimize short-input (256-bit) hashing speed.
Use sponge hash (with ChaCha12 permutation).

Use fast stream cipher (again ChaCha12).

Vectorize hash software and cipher software.

See paper for details: `sphincs.cr.yp.to`



NIST submission SPHINCS+

- Post-quantum signature based on hash functions.
- Requires only a secure hash function, no further assumptions.
- Based on ideas of Lamport (1979) and Merkle (1979).
- Developed starting from SPHINCS with
  - improve multi-signature,
  - smaller keys,
  - Option for shorter signatures (30kB instead of 41kB) if "only" 2^{50} messages signed.
- Three versions (using different hash functions)
  - SPHINCS+-SHA3 (with SHAKE256),
  - SPHINCS+-SHA2 (with SHA-256),
  - SPHINCS+-Haraka (with Haraka, a hash function for short inputs).

More info at https://sphincs.org/.

See also my course page for more videos and slides for hash-based signatures and more PQC.