Cryptanalysis of NISTPQC submissions

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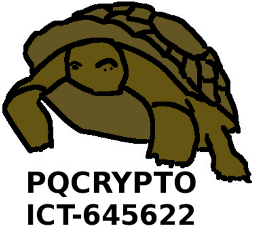

18 August 2018

Workshops on Attacks in Cryptography

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August 11, 2015

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Post-quantum cryptography

- 2015 Finally even NSA admits that the world needs post-quantum crypto.
- 2016 Every agency posts something (NCSC UK, NCSC NL, NSA (broken certificate!)).
- 2016 NIST announces call for submissions to post-quantum project, solicits submissions on signatures, encryption, and key exchange.

Post-quantum cryptography

- 10 years of motivating people to work on post-quantum crypto.
- 2015 Finally even NSA admits that the world needs post-quantum crypto.
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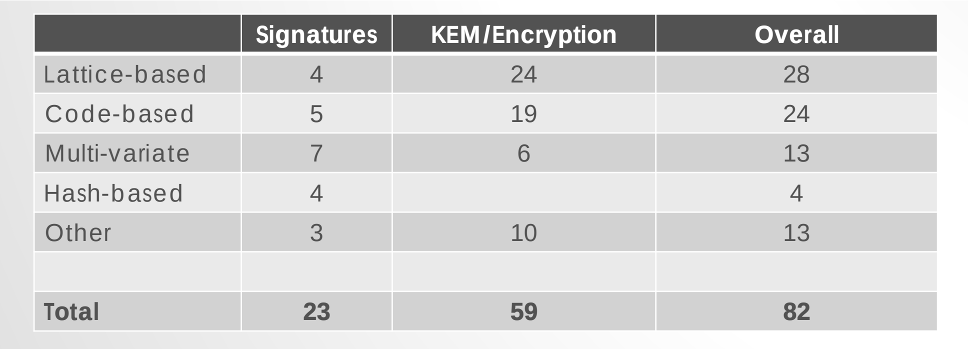
NIST Post-Quantum “Competition”

December 2016, after public feedback: NIST calls for submissions of post-quantum cryptosystems to standardize.

30 November 2017: NIST receives 82 submissions.

Overview from Dustin Moody’s (NIST) talk at Asiacrypt:



	Signatures	KEM/Encryption	Overall
Lattice-based	4	24	28
Code-based	5	19	24
Multi-variate	7	6	13
Hash-based	4		4
Other	3	10	13
Total	**23**	**59**	**82**

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Attack timeline: month 0

2017.12.18 Bernstein–Groot Bruinderink–Panny–Lange:
attack script breaking CCA for HILA5

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☢: submitter has claimed patent on submission.
Warning: Other people could also claim patents.

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Attack timeline: subsequent events

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Color coding: **total break**; **partial break**

PQCRYPTO
ICT-645622

HILA5

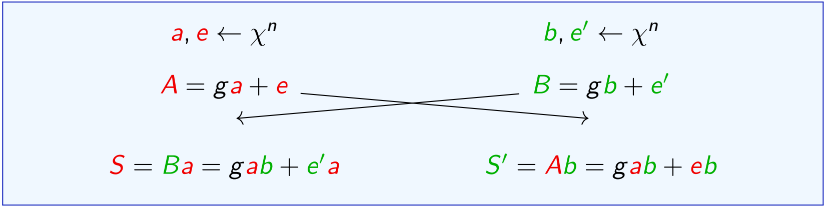
- HILA5 is a RLWE-based KEM submitted to NISTPQC.

  > *This design also provides* ***IND-CCA secure*** *KEM-DEM public key encryption if used in conjunction with an appropriate AEAD such as NIST approved AES256-GCM.*
  >
  > — HILA5 NIST submission document (v1.0)

- Decapsulation much faster than encapsulation (and faster than any other scheme).
- No mention of a CCA transform (e.g. Fujisaki–Okamoto).

Noisy Diffie–Hellman

- Have a ring $R = \mathbf{Z}[x]/(q, \varphi)$ where $q \in \mathbf{Z}$ and $\overset{\text{degree } n}{\varphi} \in \mathbf{Z}[x]$.
- Let $\underline{\chi}$ be a narrow distribution around $0 \in R$.
- Fix some "random" element $g \in R$.

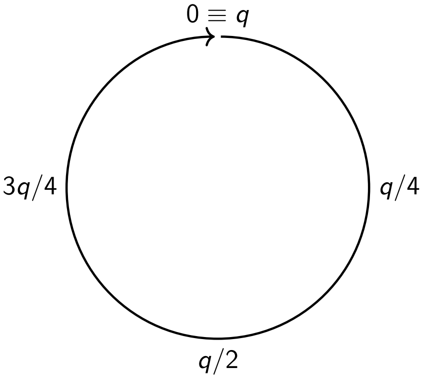


$$\begin{tikzcd}
a, e \leftarrow \chi^n & & b, e' \leftarrow \chi^n \\
A = ga + e \arrow[drr] & & B = gb + e' \arrow[dll] \\
S = Ba = gab + e'a & & S' = Ab = gab + eb
\end{tikzcd}$$

$$\Longrightarrow S - S' = e'a - eb \underset{\substack{\uparrow \\ \chi\text{ small}}}{\approx} 0$$

Reconciliation

Alice and Bob obtain close secret vectors $S, S' \in (\mathbf{Z}/q)^n$.
How to map coefficients to bits?



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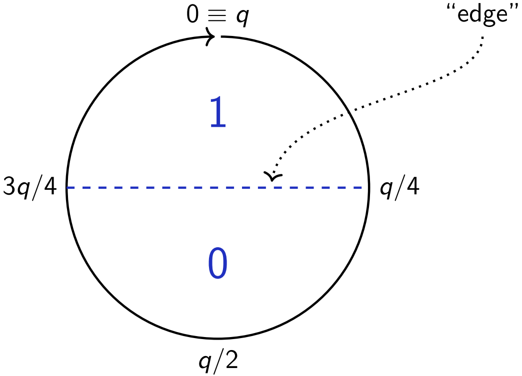

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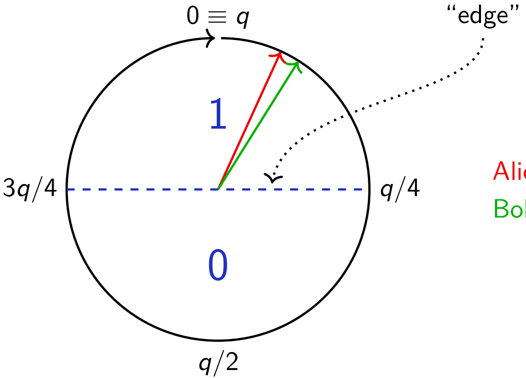

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Bob: 1

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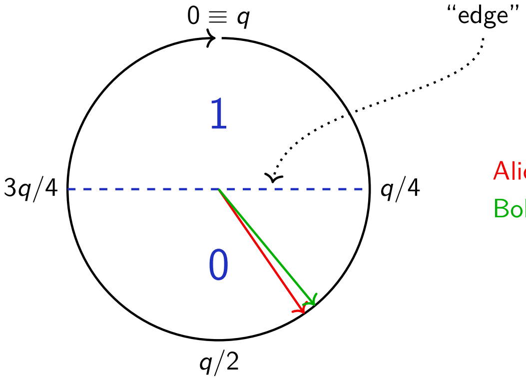

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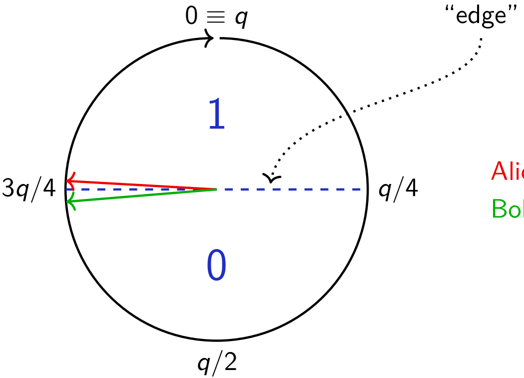

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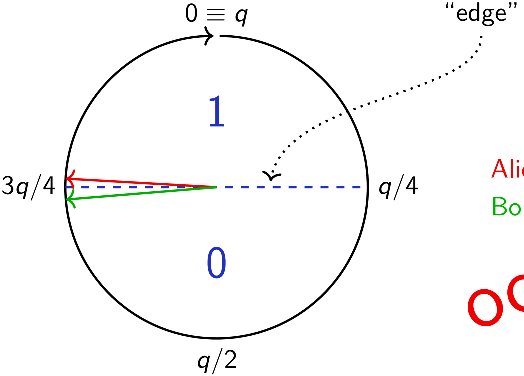

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oops!

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Mapping coefficients to bits using <u>fixed intervals</u> is <u>bad</u>.

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Better: Bob chooses a mapping based on his coefficient and tells Alice which mapping he used.

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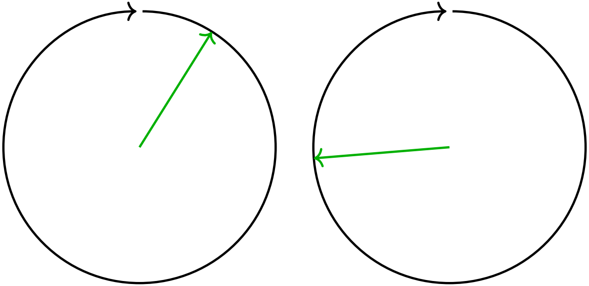

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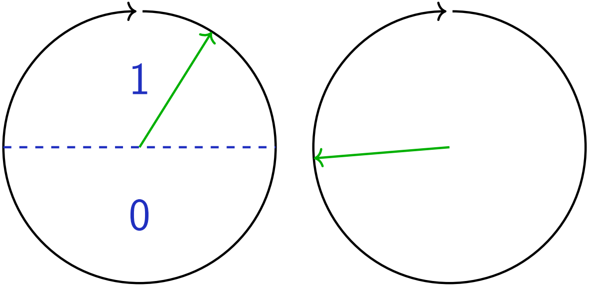

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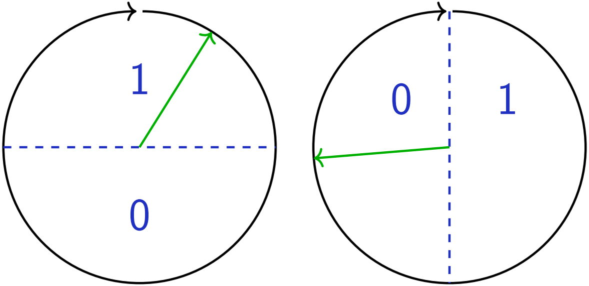

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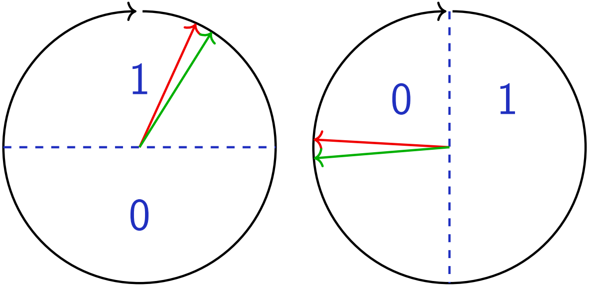

Fluhrer's attack https://ia.cr/2016/085

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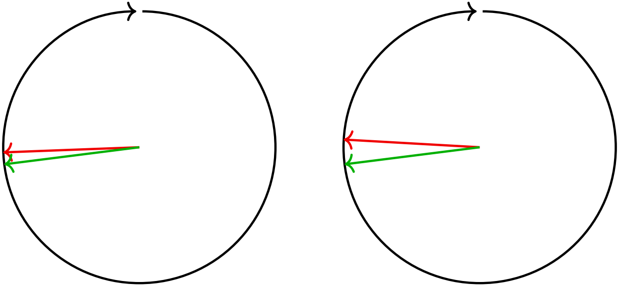

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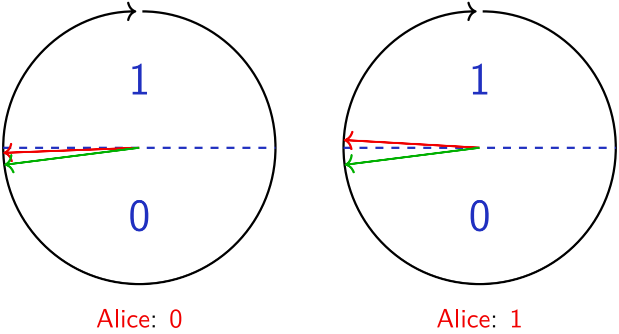

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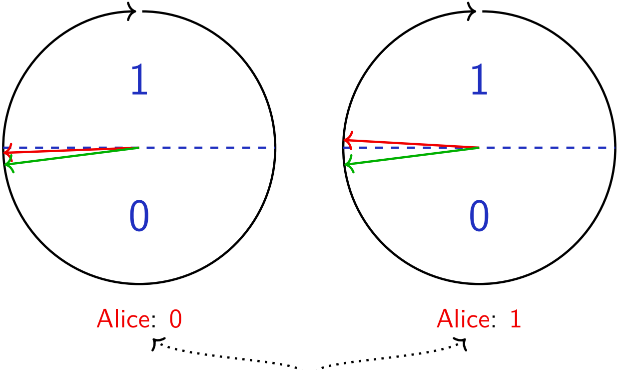

Alice: 0

Alice: 1

Evil Bob can distinguish these cases!

(He knows all the other key bits.)

Chosen-ciphertext information leaks

Evil Bob has two guesses k_0, k_1 for what Alice's key k will be given his manipulated public key B.

Alice

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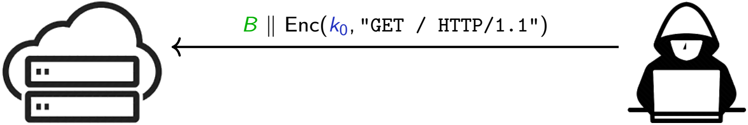

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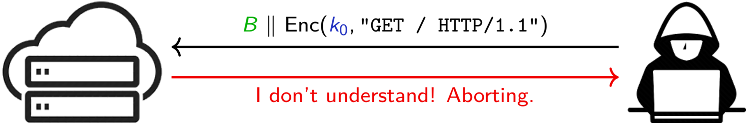

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I don't understand! Aborting.

Alice

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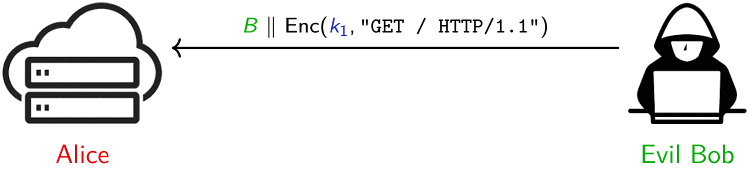

$B \parallel \mathrm{Enc}(k_1,$ `"GET / HTTP/1.1"`$)$

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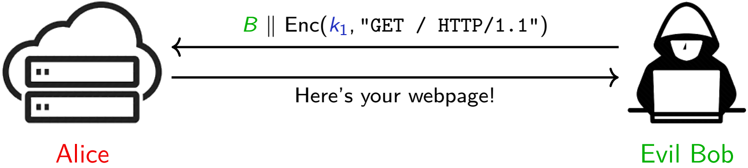

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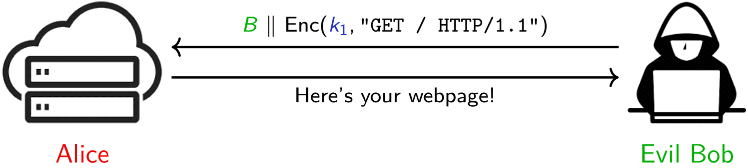

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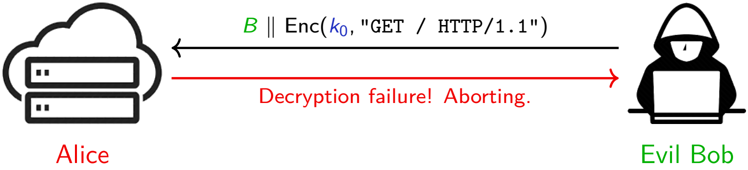

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Decryption failure! Aborting.

Alice

Evil Bob

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This still works if Enc is an authenticated symmetric cipher!

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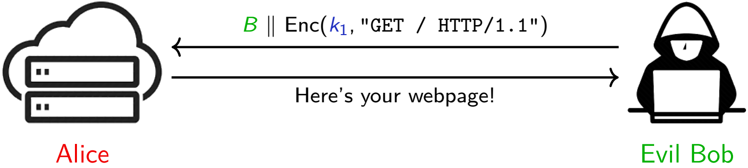

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Suppose Evil Bob knows b_δ such that $gab_\delta[0] = \overset{\text{edge}}{M} + \delta$.
$\Longrightarrow$ Querying Alice with $b = b_\delta$ leaks whether $-e'a[0] > \delta$.

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Structure of R

$\rightsquigarrow$ Can choose e' such that $e'a[0] = a[i]$ to recover all of a.

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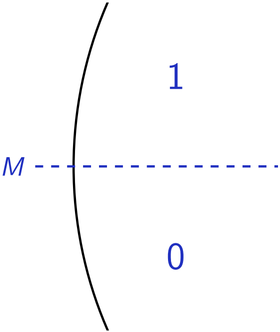
Querying Alice with $b = b_\delta$ and $e' = 1$ leaks whether $-a[0] > \delta$.



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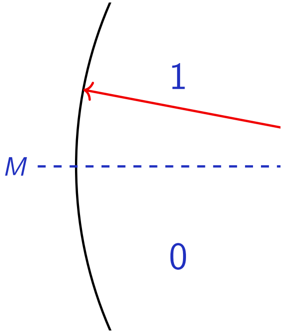

Evil Bob's δ: 0

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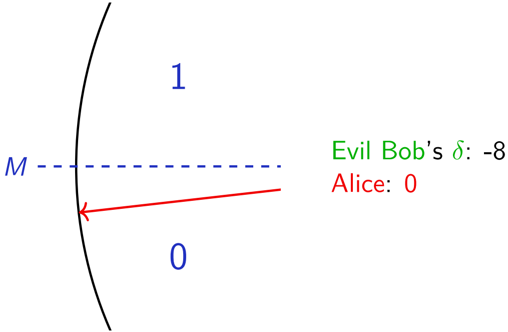

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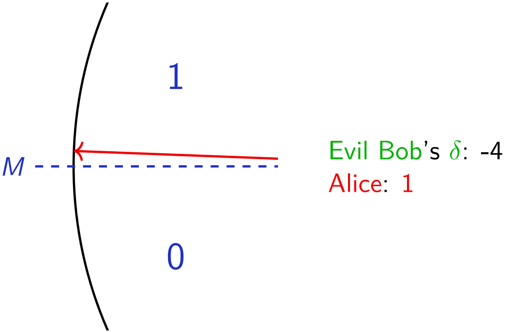

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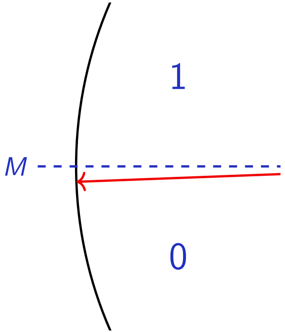

Evil Bob's δ: -6
Alice: 0

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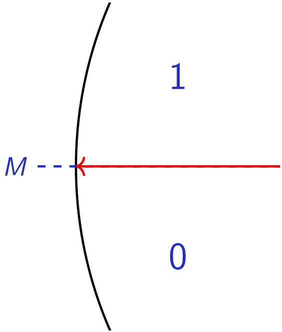

Evil Bob's δ: -5
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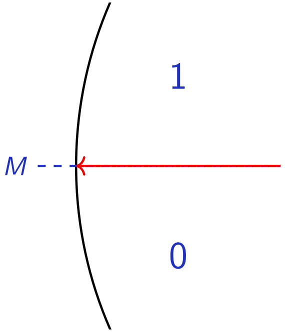

Evil Bob's δ: -5
Alice: 1

$\Longrightarrow$ Evil Bob learns that $a[0] = 5$.

Our work

Adaption of Fluhrer's attack to HILA5 and analysis

HILA5 https://ia.cr/2017/424 https://github.com/mjosaarinen/hila5

- Standard noisy Diffie–Hellman with new reconciliation.

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- Ring: $\mathbf{Z}[x]/(q, x^{1024}+1)$ where $q = 12289$.[1]
- Noise distribution χ: Ψ_{16}.[1]  on $\{-16, ..., 16\}$

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- Ring: $\mathbf{Z}[x]/(q, x^{1024}+1)$ where $q = 12289$.[1]
- Noise distribution χ: Ψ_{16}.[1]  on $\{-16, \ldots, 16\}$
- New reconciliation mechanism:
  - Only use "safe bits" that are far from an edge.
  - Additionally apply an error-correcting code.

---

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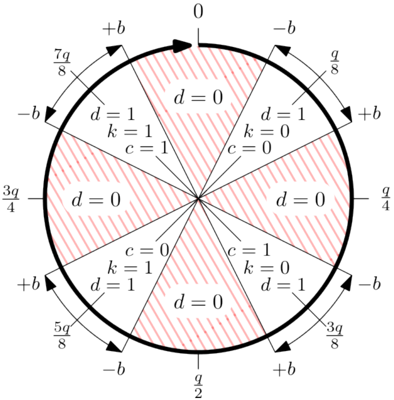
HILA5's reconciliation



(picture: HILA5 documentation)

For each coefficient:

$d = 0$: Discard coefficient.

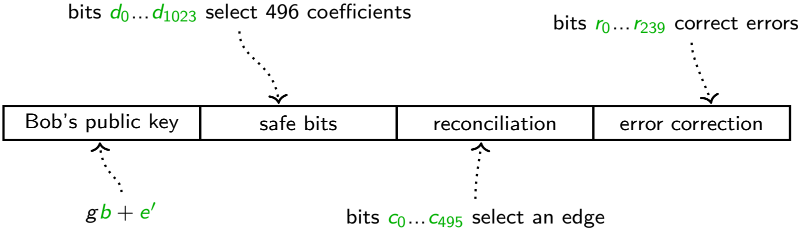
$d = 1$: Send reconciliation information c; use for key bit k.

Edges:

$c = 0$: $\lceil 3q/8 \rfloor \ldots \lceil 7q/8 \rfloor \rightsquigarrow k = 0.$
$\lceil 7q/8 \rfloor \ldots \lceil 3q/8 \rfloor \rightsquigarrow k = 1.$

$c = 1$: $\lceil q/8 \rfloor \ldots \lceil 5q/8 \rfloor \rightsquigarrow k = 0.$
$\lceil 5q/8 \rfloor \ldots \lceil \ q/8 \rfloor \rightsquigarrow k = 1.$

HILA5's packet format



bits $d_0 \ldots d_{1023}$ select 496 coefficients

bits $r_0 \ldots r_{239}$ correct errors

Bob's public key	safe bits	reconciliation	error correction

$g b + e'$

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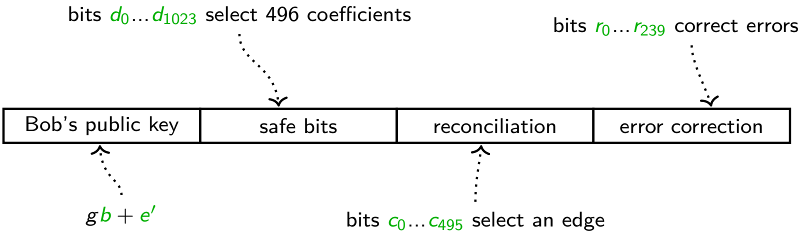

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We're going to manipulate each of these parts.

Unsafe bits

$gb + e'$	safe bits	reconciliation	error correction

We want to attack the first coefficient.

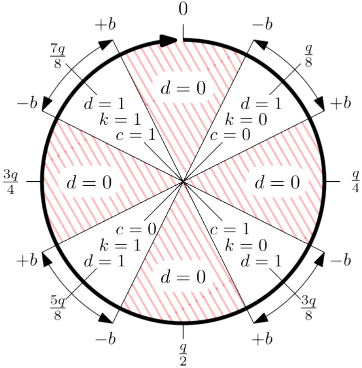
Unsafe bits

We want to attack the first coefficient.
$\Longrightarrow$ Force $d_0 = 1$ to make Alice use it.

Living on the edge

$gb + e'$	safe bits	reconciliation	error correction

We want to attack the edge at $M = \lceil q/8 \rfloor$.



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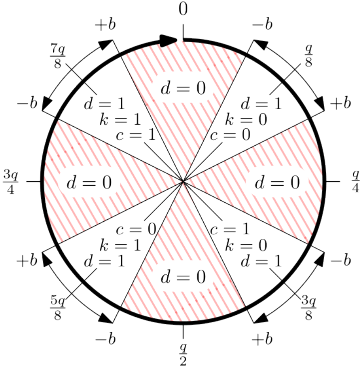
We want to attack the edge at $M = \lceil q/8 \rfloor$. $\Longrightarrow$ Force $c_0 = 1$.



Making errors

$gb + e'$	safe bits	reconciliation	error correction

- HILA5 uses a custom linear error-correcting code XE5.
- Encrypted (XOR) using part of Bob's shared secret S'.
- Ten variable-length codewords $R_0 \ldots R_9$.
- Alice corrects $S[0]$ using the first bit of each R_i.
- Capable of correcting (at least) 5-bit errors.

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We want to keep errors in $S[0]$. $\implies$ Flip the first bit of $R_0 \ldots R_4$!

All coefficients for the price of one

Our binary search recovers $e'a[0]$ from $gab_\delta + e'a$ by varying δ.
How to get $a[1]$, $a[2]$, ..?

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$gb + e'$ | safe bits | reconciliation | error correction

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By construction of $R = \mathbf{Z}[x]/(q, x^{1024} + 1)$,
Evil Bob can rotate $a[i]$ into $e'a[0]$ by setting $e' = -x^{1024-i}$.

Running the search for all i yields all coefficients of a.

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Recall that Evil Bob needs b_δ such that $gab_\delta[0] = M + \delta$.
How to obtain b_δ without knowing a?

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If b_0 was wrong, the recovered coefficients are all 0 or -1.
$\Longrightarrow$ easily detectable.

Implementation

- Our code[1] attacks the HILA5 reference implementation.
- 100% success rate in our experiments.
- Less than 6000 queries (virtually always).

  (Note: Evil Bob could recover fewer coefficients and compute the rest by solving a lattice problem of reduced dimension.)

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[1] https://helaas.org/hila5-20171218.tar.gz


TU/E KWALITEITSGARANTIE
SMEUÏG TOT OP DE BODEM
NOF
BRON VAN BOUWSTOFFEN*
NOF
HILA5
PINDAKAAS
STUKJES PINDA

HK17

"HK17 consists broadly in a Key Exchange Protocol (KEP) based on non-commutative algebra of hypercomplex numbers limited to quaternions and octonions. In particular, this proposal is based on non-commutative and non-associative algebra using octonions."

Security analysis: ". . . In our protocol, we could not find any ways to proceed with any abelianization of our octonions non-associative Moufang loop [29] or reducing of the GSDP problem of polynomial powers of octonions to a finitely generated nilpotent image of the given free group in the cryptosystem and a further nonlinear decomposition attack. We simply conclude that Roman'kov attacks do not affect our proposal."

What are octonions?

R: set of real numbers.

C: set of complex numbers; dim-2 **R**-vector space.

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Simple unified definition from 1919 Dickson:

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Exercise: Every $q \in \mathbf{O}$ has $q^2 = tq - n$ and $q^* = t - q$ for some $t, n \in \mathbf{R}$.

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Use integers modulo prime p instead of real numbers.
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Alice:

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- Generate public octonions q, r; secret $a = f_0 + f_1 q + \cdots + f_{32} q^{32}$.
- Send $q, r, a^m r a^n$ to Bob.

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Bob:

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Bob:

- Generate secret integers $k, \ell, h_0, h_1, \ldots, h_{32} > 0$.
- Generate secret $b = h_0 + h_1 q + \cdots + h_{32} q^{32}$.
- Send $b^k r b^\ell$ to Alice.

Shared secret: $a^m (b^k r b^\ell) a^n = b^k (a^m r a^n) b^\ell$.

Why does HK17 work?

Does $a^m r a^n$ mean $(a^m r)a^n$, or $a^m(ra^n)$?
Does a^m mean $a(a(\cdots))$, or $((\cdots)a)a$?

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- **Flexible identity**: $x(yx) = (xy)x$.
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$a^m(b^k r b^\ell)a^n = b^k(a^m r a^n)b^\ell$ because a, b are polynomials in q.

A fast attack, and a faster attack

Remember the exercise: q^2 is a linear combination of $1, q$.
So every polynomial in q is a linear combination of $1, q$.
There are only p^2 of these combinations!

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Even faster: Attacker solves $a^m r a^n = (q+x)r(yq+z)$.
Eight equations in three variables x, y, z; linearize.
This was the second attack script: practically instantaneous.

RaCoSS – Random Code-based Signature Schemes

- System parameters: $n = 2400$, $k = 2060$.
  Random matrix $H \in \mathbf{F}_2^{(n-k)\times n}$.
- Secret key: sparse $S \in \mathbf{F}_2^{n\times n}$.
- Public key: $T = H \cdot S$. (looks pretty random).
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PQCRYPTO
ICT-645622

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RaCoSS

Implementation bug:

```
unsigned char  c[RACOSS_N];
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/* ... */

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...compares only the first 300 coefficients!
Thus, a signature with $c[0\ldots299] = 0$ is accepted for

$$\binom{2100}{3} \Big/ \binom{2400}{3} \approx 67\%$$

of all messages.

The weight-restricted hash function (`wrhf`)

- Maps to 2400-bit strings of weight 3.
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  possible outputs.
- Slow: 600 to 800 hashes per second and core.
- Expected time for a preimage on ≈ 100 cores: 10 hours.
- crashed while brute-forcing: memory leaks
- another message signed by the first KAT:

NISTPQC is so much fun! 10900qmmP

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  Pick $n - k$ columns of H that form an invertible matrix H_1.
- Compute $z = (z_1 || 00 \ldots 0)$ by linear algebra.
- Expected weight of z is $\approx (n-k)/2 = 170 \ll 1564$.
- Properly generated signatures have $\text{weight}(z) \approx 261$.

RaCoSS – Summary

- Bug in code: bit vs. byte confusion meant only every 8th bit verified.
- Preimages for RaCoSS' special hash function: only

  $$\binom{2400}{3} = 2301120800 \sim 2^{31.09}$$

  possible outputs.
- The code dimensions give a lot of freedom to the attacker – our forged signature is better than a real one!

Code-based encryption

BIG QUAKE
Classic McEliece
LAKE
LOCKER
DAGS
LEDAkem
LEDApkc
Lepton
McNie

Edon-K✝
BIKE☢
HQC☢
NTS-KEM☢
Ouroboros-R☢
QC-MDPC KEM☢
RQC☢
RLCE-KEM☢

✝: submitter has withdrawn submission.

Lattice-based encryption

CRYSTALS-KYBER
EMBLEM and R.EMBLEM
FrodoKEM
KINDI
LAC
LIMA
LOTUS
NewHope
NTRUEncrypt
NTRU-HRSS-KEM

NTRU Prime
Odd Manhattan
SABER
Titanium
HILA5
Ding Key Exchange☢
Lizard☢
KCL OKCN/AKCN/CNKE☢
Round2☢
Compact LWE☢

Other encryption

SIKE: isogeny-based encryption

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Mersenne-756839: integer-ring encryption
Ramstake: integer-ring encryption
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SRTPI✝: multivariate encryption
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Guess Again: hard to classify
HK17✝: hard to classify
RVB✝: hard to classify

Signatures

Gravity-SPHINCS: hash-based
Picnic: hash-based
SPHINCS+: hash-based

DualModeMS: multivariate
GeMSS: multivariate
HiMQ-3: multivariate
LUOV: multivariate
Giophantus: multivariate
Gui☢: multivariate
MQDSS☢: multivariate
Rainbow☢: multivariate

pqRSA: factoring-based

CRYSTALS-DILITHIUM: lattice-based
qTESLA: lattice-based
DRS: lattice-based
FALCON☢: lattice-based
pqNTRUSign☢: lattice-based

pqsigRM: code-based
RaCoSS: code-based
RankSign✝: code-based

WalnutDSA☢: braid-group

Further resources

- https://2017.pqcrypto.org/school: PQCRYPTO summer school with 21 lectures on video + slides + exercises.
- https://2017.pqcrypto.org/exec: Executive school (12 lectures), less math, more overview. So far slides, soon videos.
- https://pqcrypto.org: Our survey site.
  - Many pointers: e.g., to PQCrypto conferences.
  - Bibliography for 4 major PQC systems.
- https://pqcrypto.eu.org: PQCRYPTO EU project.
  - Expert recommendations.
  - Free software libraries.
  - More video presentations, slides, papers.
- https://twitter.com/pqc_eu: PQCRYPTO Twitter feed.
- https://twitter.com/PQCryptoConf: PQCrypto conference Twitter feed.
- https://csrc.nist.gov/projects/post-quantum-cryptography/round-1-submissions NIST PQC competition.